Application No 10/042,908
METHOD AND APPARATUS FOR SOLVING SYSTEMS OF NONLINEAR EQUATIONS USING INTERVAL ARITHMETIC
Inventor(s) Walster et al.

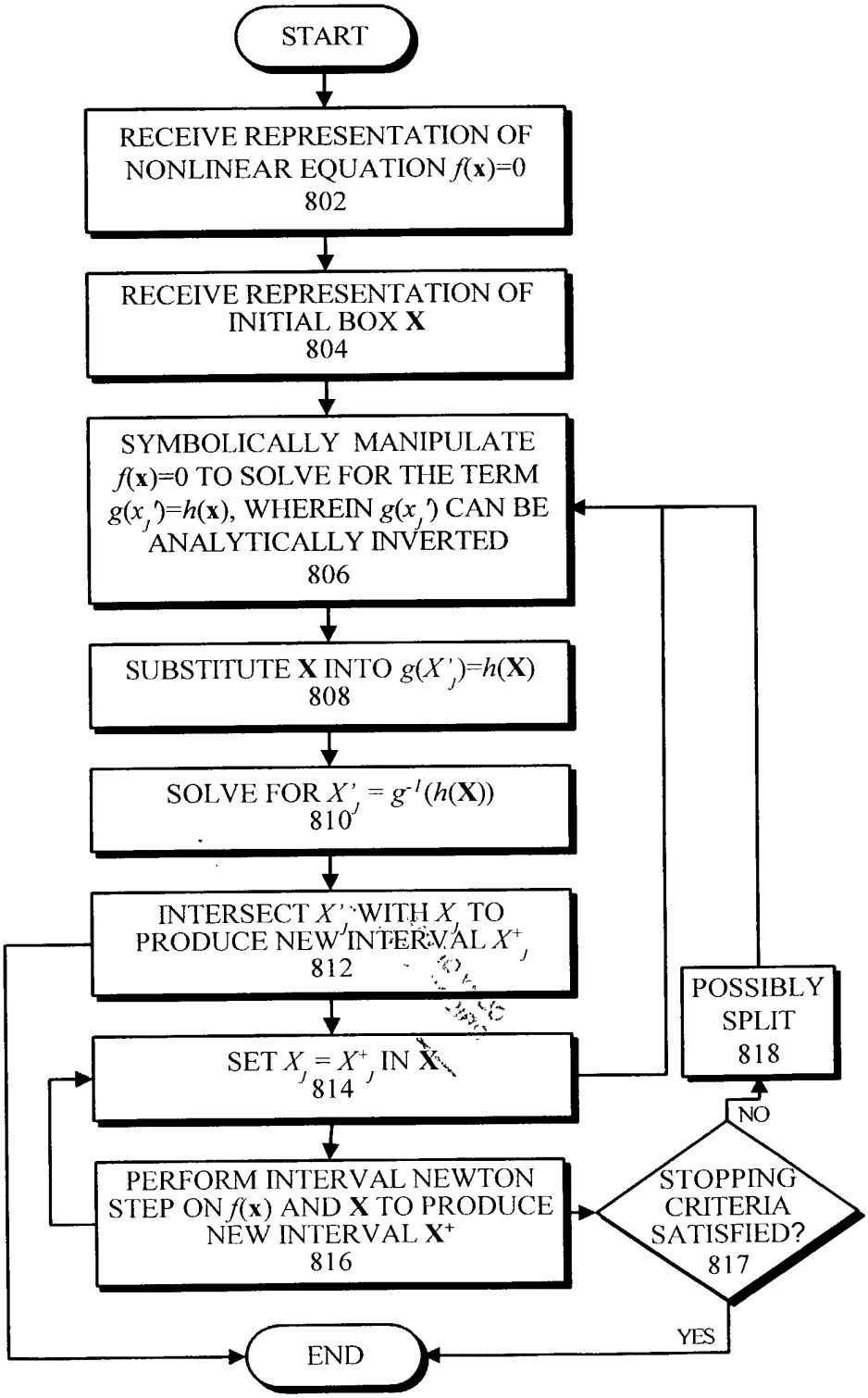

FIG. 8

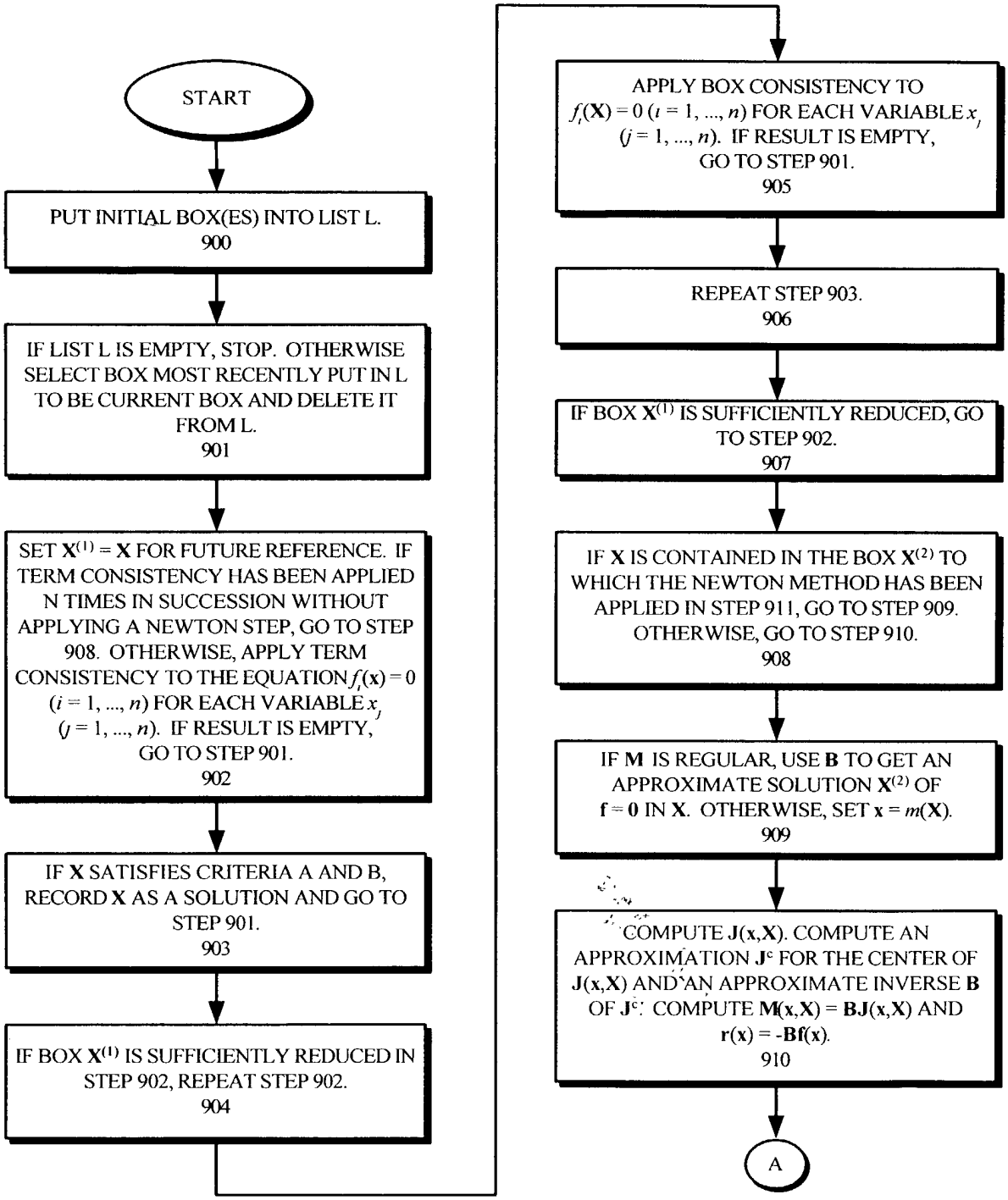
START
PUT INITIAL BOX(ES) INTO LIST L.
900
IF LIST L IS EMPTY, STOP. OTHERWISE SELECT BOX MOST RECENTLY PUT IN L TO BE CURRENT BOX AND DELETE IT FROM L.
901
SET $\mathbf{X}^{(1)} = \mathbf{X}$ FOR FUTURE REFERENCE. IF TERM CONSISTENCY HAS BEEN APPLIED N TIMES IN SUCCESSION WITHOUT APPLYING A NEWTON STEP, GO TO STEP 908. OTHERWISE, APPLY TERM CONSISTENCY TO THE EQUATION $f_i(\mathbf{x}) = 0$ ($i = 1, ..., n$) FOR EACH VARIABLE x_j ($j = 1, ..., n$). IF RESULT IS EMPTY, GO TO STEP 901.
902
IF **X** SATISFIES CRITERIA A AND B, RECORD **X** AS A SOLUTION AND GO TO STEP 901.
903
IF BOX $\mathbf{X}^{(1)}$ IS SUFFICIENTLY REDUCED IN STEP 902, REPEAT STEP 902.
904
APPLY BOX CONSISTENCY TO $f_i(\mathbf{X}) = 0$ ($i = 1, ..., n$) FOR EACH VARIABLE x_j ($j = 1, ..., n$). IF RESULT IS EMPTY, GO TO STEP 901.
905
REPEAT STEP 903.
906
IF BOX $\mathbf{X}^{(1)}$ IS SUFFICIENTLY REDUCED, GO TO STEP 902.
907
IF **X** IS CONTAINED IN THE BOX $\mathbf{X}^{(2)}$ TO WHICH THE NEWTON METHOD HAS BEEN APPLIED IN STEP 911, GO TO STEP 909. OTHERWISE, GO TO STEP 910.
908
IF **M** IS REGULAR, USE **B** TO GET AN APPROXIMATE SOLUTION $\mathbf{X}^{(2)}$ OF $\mathbf{f} = \mathbf{0}$ IN **X**. OTHERWISE, SET $\mathbf{x} = m(\mathbf{X})$.
909
COMPUTE $\mathbf{J}(\mathbf{x},\mathbf{X})$. COMPUTE AN APPROXIMATION $\mathbf{J}^c$ FOR THE CENTER OF $\mathbf{J}(\mathbf{x},\mathbf{X})$ AND AN APPROXIMATE INVERSE **B** OF $\mathbf{J}^c$. COMPUTE $\mathbf{M}(\mathbf{x},\mathbf{X}) = \mathbf{BJ}(\mathbf{x},\mathbf{X})$ AND $\mathbf{r}(\mathbf{x}) = -\mathbf{Bf}(\mathbf{x})$.
910
A

FIG. 9A

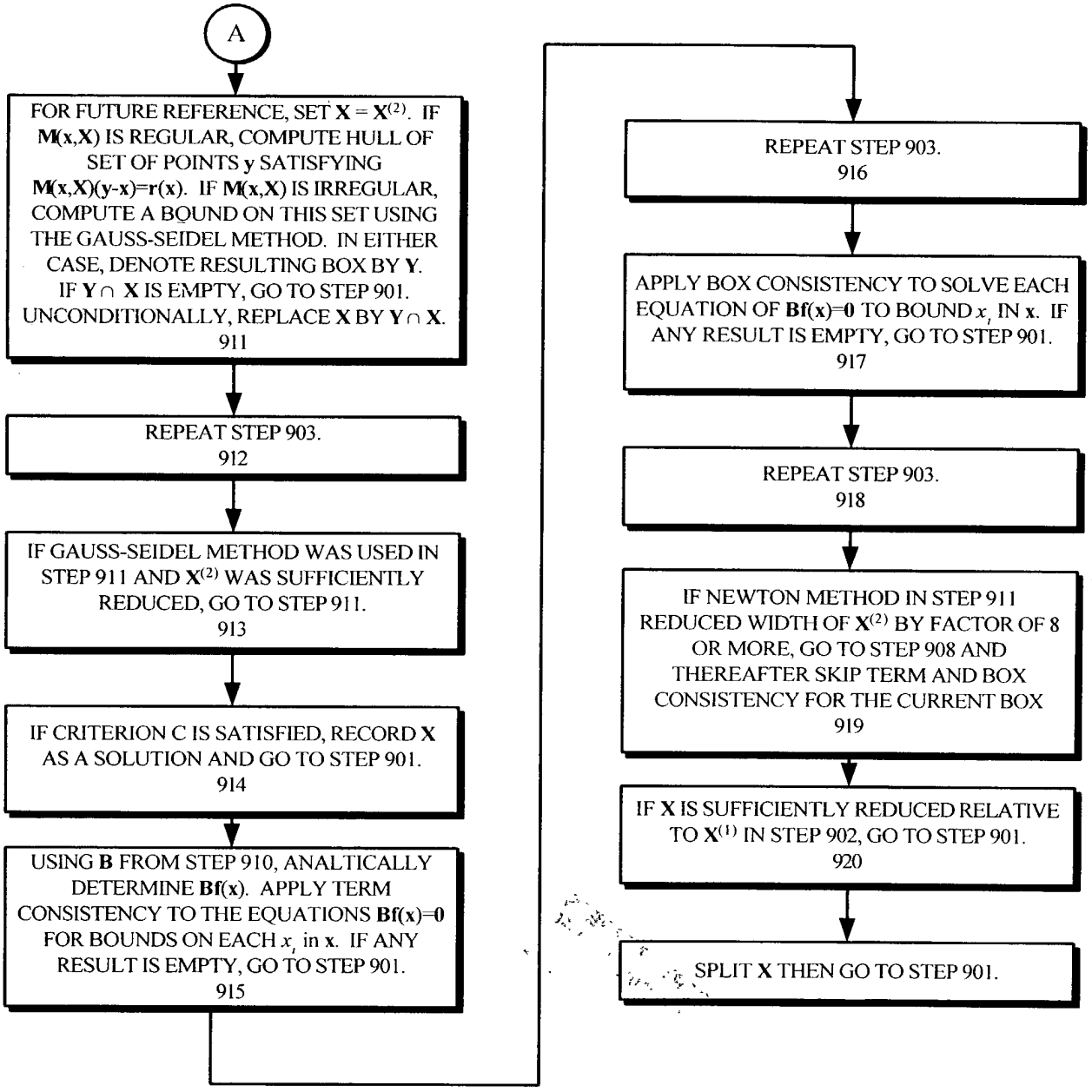
A
FOR FUTURE REFERENCE, SET X = X(2). IF M(x,X) IS REGULAR, COMPUTE HULL OF SET OF POINTS y SATISFYING M(x,X)(y-x)=r(x). IF M(x,X) IS IRREGULAR, COMPUTE A BOUND ON THIS SET USING THE GAUSS-SEIDEL METHOD. IN EITHER CASE, DENOTE RESULTING BOX BY Y. IF Y ∩ X IS EMPTY, GO TO STEP 901. UNCONDITIONALLY, REPLACE X BY Y ∩ X.
911
REPEAT STEP 903.
912
IF GAUSS-SEIDEL METHOD WAS USED IN STEP 911 AND X(2) WAS SUFFICIENTLY REDUCED, GO TO STEP 911.
913
IF CRITERION C IS SATISFIED, RECORD X AS A SOLUTION AND GO TO STEP 901.
914
USING B FROM STEP 910, ANALTICALLY DETERMINE Bf(x). APPLY TERM CONSISTENCY TO THE EQUATIONS Bf(x)=0 FOR BOUNDS ON EACH x_i in x. IF ANY RESULT IS EMPTY, GO TO STEP 901.
915
REPEAT STEP 903.
916
APPLY BOX CONSISTENCY TO SOLVE EACH EQUATION OF Bf(x)=0 TO BOUND x_i IN x. IF ANY RESULT IS EMPTY, GO TO STEP 901.
917
REPEAT STEP 903.
918
IF NEWTON METHOD IN STEP 911 REDUCED WIDTH OF X(2) BY FACTOR OF 8 OR MORE, GO TO STEP 908 AND THEREAFTER SKIP TERM AND BOX CONSISTENCY FOR THE CURRENT BOX
919
IF X IS SUFFICIENTLY REDUCED RELATIVE TO X(1) IN STEP 902, GO TO STEP 901.
920
SPLIT X THEN GO TO STEP 901.

FIG. 9B